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Some Notes on Generalized Young Inequality for n Numbers

*Corresponding email: [1] XXXXXXXXX

Abstract. In this note we obtain a generalized of Young's inequality for n numbers. From the inequality, we also get the generalized of Ho ̈lder's Inequality and Minkowski's inequality for n terms. Furthermore result, some improved of the generalized Young's inequality for n numbers is discussed.

Keywords: Young's Inequality, Holder's Inequality, Minkowski's inequality

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1. Introduction

Let a and b be positive numbers. The Famous Young inequality [1] state that

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q} \tag{1.1}$$

for every $p, q \geq 1$ with $\frac{1}{p} + \frac{1}{q} = 1$ or $q = \frac{p}{p-1}$.

The Classical Young inequality is also rewritten as

$$a^{\lambda} b^{1-\lambda} \leq \lambda a + (1-\lambda) b \tag{1.2}$$

by putting $a := a^{\frac{1}{p}}$, $b := b^{\frac{1}{q}}$, and $\lambda = \frac{1}{p}$ (clearly, $0 \leq \lambda \leq 1$).

Based on Young's inequality, we can derive two other well-known inequalities namely the Holder's inequality and the Minkowski's inequality, which are some applications of Young's inequality. Apart from these two inequalities, Young's inequality also has many other applications. So, many mathematicians are interested in discussing Young's inequality.

Many researchers also try to generalize, improve, and refine these inequalities. An improvement of Young's inequality, obtained by F. Kittaneh and Y. Manasrah [2], is as follows

$$a^{\lambda}b^{1-\lambda} + r_0\left(\sqrt{a} - \sqrt{b}\right)^2 \le \lambda a + (1-\lambda)b \tag{1.3}$$

with $r_0 = \min\{\lambda, 1-\lambda\}$.

The authors of [3] obtained another refinement of the Young inequality as

$$\left(a^{\lambda}b^{1-\lambda}\right)^2 + r_0^2(a-b)^2 \le (\lambda a + (1-\lambda)b)^2 \tag{1.4}$$

with $r_0 = \min\{\lambda, 1-\lambda\}$.

Other development results of Young's inequality can also be seen in [4-9].

In the other hand, we know that the classical Young inequality for two scalars is the ν-weighted arithmetic–geometric mean inequality, which is a fundamental relation between two nonnegative real numbers. The question arises: what is the generalization of Young's inequality if there are n numbers? as an application of the generalization of these inequalities, what inequalities can be generated? And what is the form of development of the generalization of Young's inequality?

In this paper, we will introduce the generalization of Young's inequalities involving n numbers. This paper will also discuss the inequalities that will result from the generalization of these inequalities, in particular the generalization of Hölder's inequality and generalization of Minkowski's inequality. And furthermore, inspired by the refinement of [2] and [3], we will also discuss the refinement of the generalization of these inequalities.

2. Main Result

2.1. Generalized Young's inequality for n numbers

We will start by introducing a generalization of Young's inequality involving n numbers. This form is a generalization of the classical form of Young's inequality.

Theorem 1. If a_i are positive numbers for all $i = 1,2,3,\dots,n$ and $p_i > 1$ for all $i = 1,2,3,\dots,n$ such that $\sum_{k=1}^{n}\frac{1}{p_k} = 1$, then

$$\prod_{i=1}^{n} a_i \le \sum_{i=1}^{n}\frac{1}{p_i}(a_i)^{p_i} \tag{2.1}$$

Proof:

We will use mathematical induction to prove this theorem. For $n = 1$, the inequality is clearly satisfied. For $n = 2$, the inequality is directly satisfied from Young's inequality.

Now, assume that the inequality is true for $n = k$. we get

$$\prod_{i=1}^{k} a_i \le \sum_{i=1}^{k}\frac{1}{p_i}(a_i)^{p_i}$$

The next, we will proof that the inequality is also true for $n = k+1$.

$$\prod_{i=1}^{k+1} a_i = a_1 \cdot a_2 \cdot \ldots \cdot a_{k-1} \cdot (a_k a_{k+1}) \le \sum_{k=1}^{k-1}\frac{1}{p_i}(a_i)^{p_i} + \frac{1}{p_k}(a_k a_{k+1})^{p_k}$$

$$= \sum_{k=1}^{k-1} \frac{1}{p_i} (a_i)^{p_i} + \frac{1}{p_k} (a_k)^{p_k} (a_{k+1})^{p_k}$$

$$\leq \sum_{k=1}^{k-1} \frac{1}{p_i} (a_i)^{p_i} + \frac{1}{p_k} \left[\left(\frac{((a_k)^{p_k})^{\frac{q}{p_k}}}{\frac{q}{p_k}} \right) + \left(\frac{((a_{k+1})^{p_k})^{\frac{q}{q-p_k}}}{\frac{q}{q-p_k}} \right) \right]$$

$$= \sum_{k=1}^{k-1} \frac{1}{p_i} (a_i)^{p_i} + \frac{1}{p_k} \left[\left(\frac{(a_k)^q}{\frac{q}{p_k}} \right) + \left(\frac{(a_{k+1})^{\frac{p_k q}{q-p_k}}}{\frac{q}{q-p_k}} \right) \right]$$

$$= \sum_{k=1}^{k-1} \frac{1}{p_i} (a_i)^{p_i} + \frac{(a_k)^q}{q} + \frac{(a_{k+1})^{\frac{p_k q}{q-p_k}}}{\frac{p_k q}{q-p_k}}$$

$$= \sum_{k=1}^{k+1} \frac{1}{q_i} (a_i)^{q_i}$$

where $q_i = p_i$ for $i = 1, \dots, k-1$, $q_k = q$, and $q_{k+1} = \frac{p_k q}{q - p_k}$. Furthermore, we can easily prove that $\sum_{k=1}^{k+1} \frac{1}{q_i} = 1$. So, the inequality has been proven.

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Next, the inequality (2.1) can be written as

$$\prod_{i=1}^{n} (a_i)^{\lambda_i} \leq \sum_{i=1}^{n} \lambda_i a_i \tag{2.2}$$

Remark. When comparing the inequality (2.1) with the inequality (1.1), or (1.2) with (2.2), it is easy to observe that the left-hand side and the righthand side in the inequality (2.1) or (2.2) consist of n numbers, while in the inequality (1.1) or (1.2) there are only 2 numbers. It should be noticed here that either the inequality (2.1) or (2.2) is a generalization of the inequality (1.1) or (1.2).

2.2. Generalized Hölder's inequality with n terms

In this chapter, we will introduce a generation of Hölder's inequalities obtained from the generalized Young's inequalities for n numbers.

Theorem 2.2. For any vectors x_i in $\mathbb{C}^m$, and for any positif numbers p_i satisfying $\sum_{i=1}^{n} \frac{1}{p_i} = 1$, we have

$$\sum_{k=1}^{m} \left| \prod_{i=1}^{n} x_{i_k} \right| \leq \prod_{i=1}^{n} \|x_i\|_{p_i}, \tag{2.3}$$

where

$$\|x_i\|_{p_i} = \left(\sum_{k=1}^{m} |x_{i_k}|^{p_i} \right)^{\frac{1}{p_i}}.$$

Proof:

If one of x_i is zero, the inequality certainly holds with equality. Otherwise, assume x_i are nonzero for every $i \in \{1,2,3,\dots,n\}$, and let $u_i = \frac{x_i}{\|x_i\|_{p_i}}$, and note that $\|u_i\|_{p_i} = 1$ for all i. Then by using equation (2.1)

$$\begin{aligned}
\sum_{k=1}^{m}\left|\prod_{i=1}^{n} u_{i_k}\right| &= \sum_{k=1}^{m}\left(\prod_{i=1}^{n}|u_{i_k}|\right)\\
&\le \sum_{k=1}^{m}\left(\sum_{i=1}^{n}\frac{1}{p_i}|u_{i_k}|^{p_i}\right)\\
&= \sum_{i=1}^{n}\left(\sum_{k=1}^{m}\frac{1}{p_i}|u_{i_k}|^{p_i}\right)\\
&= \sum_{i=1}^{n}\left(\frac{1}{p_i}\sum_{k=1}^{m}|u_{i_k}|^{p_i}\right)\\
&= \sum_{i=1}^{n}\left(\frac{1}{p_i}(\|u_i\|_{p_i})^{p_i}\right)\\
&= \sum_{i=1}^{n}\frac{1}{p_i}\\
&= 1.
\end{aligned}$$

Now multiplying both sides by the positive quantity $\prod_{i=1}^{n}\|x_i\|_{p_i}$ to obtain the statement of the theorem. To achieve equality, each term in the sum must achieve equality in inequality (2.1), i.e., for all $k \in \{1,2,3,\dots,m\}$, $|u_{1_k}| = |u_{2_k}| = \cdots = |u_{n_k}|$, wich translates to the statement in the theorem since $|u_i| = \frac{|x_i|}{\|x_i\|_{p_i}}$ for all i.

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Remark. The inequality (2.3) is a generalization of the Hölder inequality which is known as follows. For any vectors x and y in $\mathbb{C}^m$, and for any positif numbers p and q satisfying $\frac{1}{p} + \frac{1}{q} = 1$, we have

$$\sum_{k=1}^{m}|x_k + y_k| \le \|x\|_p\|x\|_q,$$

Where $\|x\|_p = (\sum_{k=1}^{m}|x_k|^p)^{\frac{1}{p}}$ dan $\|y\|_p = (\sum_{k=1}^{m}|y_k|^q)^{\frac{1}{q}}$.

2.3. Generalized Minkowski's inequality with n terms

In this chapter, we will also introduce a generation of Minkowski's inequalities.

Theorem 2.3. For any vectors u_i in $\mathbb{C}^m$, and for any positive number $p > 1$, we have

$$\left\|\sum_{i=1}^{n} u_i\right\|_p \leq \sum_{i=1}^{n} \|u_i\|_p . \tag{2.4}$$

Equality hold if and only if $au_i = bu_j$ for every $i \neq j$ and for some non-negative real constants a and b, not both zero.

Proof:

$$\begin{aligned}
\left(\left\|\sum_{i=1}^{n} u_i\right\|_p\right)^p &= \sum_{k=1}^{m}\left|\sum_{i=1}^{n} u_{i_k}\right|^p \\
&= \sum_{k=1}^{m}\left(\left|\sum_{i=1}^{n} u_{i_k}\right| \cdot \left|\sum_{i=1}^{n} u_{i_k}\right|^{p-1}\right) \\
&\leq \sum_{k=1}^{m}\left(\left(\sum_{i=1}^{n}|u_{i_k}|\right) \cdot \left|\sum_{i=1}^{n} u_{i_k}\right|^{p-1}\right) \\
&= \sum_{i=1}^{n}\left(\sum_{k=1}^{m}\left(|u_{i_k}|\left|\sum_{i=1}^{n} u_{i_k}\right|^{p-1}\right)\right) \\
&\leq \sum_{i=1}^{n}\left(\sum_{k=1}^{m}|u_{i_k}|^p\right)^{\frac{1}{p}}\left(\sum_{k=1}^{m}\left(\left|\sum_{i=1}^{n} u_{i_k}\right|^{p-1}\right)^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}} \\
&= \left(\sum_{i=1}^{n}\left\|\sum_{i=1}^{n} u_i\right\|_p\right)\left(\left\|\sum_{i=1}^{n} u_i\right\|_p\right)^{p-1}.
\end{aligned}$$

The theorem follows by dividing both sides by the positive quantity $\left(\|\sum_{i=1}^{n} u_i\|_p\right)^{p-1}$. To achieve equality it is necessary that the triangle inequality (n numbers) for complex numbers holds with equality for each term.

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Remark. The inequality (2.4) is a generalization of the holder inequality which is known as follows. For any vectors u and v in $\mathbb{C}^m$, and for any positif number $p > 1$, we have

$$\|u + v\|_p \leq \|u\|_p + \|v\|_p.$$

Equality hold if and only if $au = bv$ for some non-negative real constants a and b, not both zero.

3. Refinements of the scalar Young inequality

Theorem 3.1. If a_i are positive numbers for all $i = 1,2,3, \dots, n$ and $0 \leq \lambda_i \leq 1$ for all $i = 1,2,3, \dots, n$ such that $\sum_{i=1}^{n} \lambda_i = 1$, then

$$\prod_{i=1}^{n} a_i^{\lambda_i} + \sum_{i=1}^{n-1} r_i\left(\sqrt{a_i} - \sqrt{a_n}\right)^2 \le \sum_{i=1}^{n} \lambda_i a_i \tag{3.3}$$

where $r_i = \min\left\{\lambda_i, 1 - \lambda_i - 2\sum_{\substack{j=1 \\ j \ne i}}^{n-1} \lambda_j\right\}$ for $i \in \{1,2,3,\dots,n\}$.

Proof:

If $\lambda_i = \frac{1}{2}$ for every i and $n = 2$, the inequality becomes an equality. Assume that $\lambda_i < \frac{1}{2}$ for every $i \in \{1,2,3,\dots,n\}$. Then by the inequality, we have

$$\begin{aligned}
\sum_{i=1}^{n} \lambda_i a_i - \sum_{i=1}^{n-1} \lambda_i\left(\sqrt{a_i} - \sqrt{a_n}\right)^2 &= \sum_{i=1}^{n} \lambda_i a_i - \sum_{i=1}^{n-1} \lambda_i\left(a_i - 2\sqrt{a_i a_n} + a_n\right) \\
&= \lambda_n a_n + \sum_{i=1}^{n-1} 2\lambda_i \sqrt{a_i a_n} - \sum_{i=1}^{n-1} \lambda_i a_n \\
&= \sum_{i=1}^{n-1} 2\lambda_i \sqrt{a_i a_n} + \left(\lambda_n a_n - \sum_{i=1}^{n-1} \lambda_i a_n\right) \\
&= \sum_{i=1}^{n-1} 2\lambda_i \sqrt{a_i a_n} + \left(\lambda_n - \sum_{i=1}^{n-1} \lambda_i\right) a_n \\
&= \sum_{i=1}^{n-1} 2\lambda_i \sqrt{a_i a_n} + \left(1 - 2\sum_{i=1}^{n-1} \lambda_i\right) a_n \\
&\ge \left(\prod_{i=1}^{n-1} \left(\sqrt{a_i a_n}\right)^{2\lambda_i}\right) \cdot a_n^{\left(1 - 2\sum_{i=1}^{n-1} \lambda_i\right)} \\
&= \left(\prod_{i=1}^{n-1} (a_i a_n)^{\lambda_i}\right) \cdot a_n^{\left(1 - 2\sum_{i=1}^{n-1} \lambda_i\right)} \\
&= \left(\prod_{i=1}^{n-1} (a_i)^{\lambda_i}\right) \cdot a_n^{\left(1 - \sum_{i=1}^{n-1} \lambda_i\right)} \\
&= \left(\prod_{i=1}^{n} (a_i)^{\lambda_i}\right),
\end{aligned}$$

Where $\lambda_n = 1 - \sum_{i=1}^{n-1} \lambda_i$, and so

$$\sum_{i=1}^{n} \lambda_i a_i - \sum_{i=1}^{n-1} \lambda_i\left(\sqrt{a_i} - \sqrt{a_n}\right)^2 \le \prod_{i=1}^{n} a_i^{\lambda_i}$$

If there is $i_0 \in \{1,2,\dots,n-1\}$ such that $\lambda_{i_0} > \frac{1}{2}$, then $\lambda_i < \frac{1}{2}$ for every $i \in \{1,2,3,\dots,n\} - \{i_0\}$ and

$$\sum_{i=1}^{n} \lambda_i a_i - \sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i \left(\sqrt{a_i} - \sqrt{a_n}\right)^2 - (1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i)\left(\sqrt{a_{i_0}} - \sqrt{a_n}\right)^2$$

$$= \sum_{\substack{i=1 \\ i \neq i_0}}^{n} \lambda_i a_i - \sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i \left(\sqrt{a_i} - \sqrt{a_n}\right)^2 + \lambda_{i_0} a_{i_0} - \left(1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right)\left(\sqrt{a_{i_0}} - \sqrt{a_n}\right)^2$$

$$= \lambda_n a_n + \sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} 2\lambda_i \sqrt{a_i a_n} - \sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i a_n + \lambda_{i_0} a_{i_0} - \left(1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right)\left(\sqrt{a_{i_0}} - \sqrt{a_n}\right)^2$$

$$= \sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} 2\lambda_i \sqrt{a_i a_n} + \left(1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right) a_n + \lambda_{i_0} a_{i_0} - \left(1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right)\left(a_{i_0} - 2\sqrt{a_{i_0} a_n} + a_n\right)$$

$$= \sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} 2\lambda_i \sqrt{a_i a_n} + \left(2\sum_{i=1}^{n-1} \lambda_i - 1\right) a_{i_0} + 2\left(1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right)\sqrt{a_{i_0} a_n}$$

$$\geq \left(\prod_{\substack{i=1 \\ i \neq i_0}}^{n-1} \left(\sqrt{a_i a_n}\right)^{2\lambda_i}\right) \cdot a_{i_0}^{\left(2\sum_{i=1}^{n-1} \lambda_i - 1\right)} \cdot \sqrt{a_{i_0} a_n}^{\,2\left(1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right)}$$

$$= \left(\prod_{\substack{i=1 \\ i \neq i_0}}^{n-1} (a_i a_n)^{\lambda_i}\right) \cdot a_{i_0}^{\left(2\sum_{i=1}^{n-1} \lambda_i - 1\right)} \cdot (a_{i_0} a_n)^{\left(1 - \lambda_1 - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right)}$$

$$= \left(\prod_{\substack{i=1 \\ i \neq i_0}}^{n-1} (a_i)^{\lambda_i} (a_n)^{\lambda_i}\right) \cdot (a_{i_0})^{\left(2\sum_{i=1}^{n-1} \lambda_i - 1\right)} \cdot (a_{i_0})^{\left(1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right)} a_n^{\left(1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i\right)}$$

$$= \left(\prod_{\substack{i=1 \\ i \neq i_0}}^{n-1} (a_i)^{\lambda_i}\right) \cdot (a_{i_0})^{\lambda_{i_0}} \cdot a_n^{\left(1 - \sum_{i=1}^{n-1} \lambda_i\right)}.$$

$$= \left(\prod_{i=1}^{n} (a_i)^{\lambda_i}\right),$$

Where $\lambda_n = 1 - \sum_{i=1}^{n-1} \lambda_i$, and so

$$\sum_{i=1}^{n} \lambda_i a_i - \sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i \left(\sqrt{a_i} - \sqrt{a_n}\right)^2 - (1 - \lambda_{i_0} - 2\sum_{\substack{i=1 \\ i \neq i_0}}^{n-1} \lambda_i)\left(\sqrt{a_{i_0}} - \sqrt{a_n}\right)^2 \leq \prod_{i=1}^{n} a_i^{\lambda_i}$$

Hence,

$$\prod_{i=1}^{n} a_i^{\lambda_i} + \sum_{i=1}^{n-1} r_i\left(\sqrt{a_i} - \sqrt{a_n}\right)^2 \le \sum_{i=1}^{n} \lambda_i a_i$$

This completes the proof.

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As a direct consequence of Theorem 3.1, we have

$$\sum_{\substack{i_j \in \{1,2,3,\dots,n\} \\ i \neq k \to i_j \neq k_j}} \left(\prod_{i=1}^{n} a_i^{\lambda_{i_j}}\right) + n\sum_{i=1}^{n-1} r_i\left(\sqrt{a_i} - \sqrt{a_n}\right)^2 \le \sum_{i=1}^{n} a_i$$

and so

$$\frac{1}{n}\sum_{i_j \in \{1,2,3,\dots,n\}} \left(\prod_{i=1}^{n} a_i^{\lambda_{i_j}}\right) + \sum_{i=1}^{n-1} r_i\left(\sqrt{a_i} - \sqrt{a_n}\right)^2 \le \sum_{i=1}^{n} a_i .$$

Corollary 3.2. If a_i are positive numbers for all $i = 1,2,3,\dots,n$ and $0 \le \lambda_i \le 1$ for all $i = 1,2,3,\dots,n$ such that $\sum_{i=1}^{n} \lambda_i = 1$, then

$$\frac{1}{n}\sum_{i_j \in \{1,2,3,\dots,n\}} \left(\prod_{i=1}^{n} a_i^{\lambda_{i_j}}\right) + \sum_{i=1}^{n-1} r_i\left(\sqrt{a_i} - \sqrt{a_n}\right)^2 \le \sum_{i=1}^{n} a_i \qquad (3.2)$$

Corollary 3.3. If a_i are positive numbers for all $i = 1,2,3,\dots,n$ and $0 \le \lambda_i \le 1$ for all $i = 1,2,3,\dots,n$ such that $\sum_{i=1}^{n} \lambda_i = 1$, then

$$\left(\prod_{i=1}^{n} a_i^{\lambda_i}\right)^2 + \sum_{i=1}^{n-1} r_i (a_i - a_n)^2 \le \sum_{i=1}^{n} \lambda_i a_i^2 < \left(\sum_{i=1}^{n} \lambda_i a_i\right)^2 \qquad (3.3)$$

where $r_i = \min\left\{\lambda_i, 1 - \lambda_i - 2\sum_{\substack{j=1 \\ j \neq i}}^{n-1} \lambda_j\right\}$ for $i \in \{1,2,3,\dots,n\}$.

Proof:

If we putting a_i by a_i^2, the inequality can be written in the form

$$\left(\prod_{i=1}^{n} a_i^{\lambda_i}\right)^2 + \sum_{i=1}^{n-1} r_i (a_i - a_n)^2 \le \sum_{i=1}^{n} \lambda_i a_i^2 < \left(\sum_{i=1}^{n} \lambda_i a_i\right)^2$$

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Remark. When comparing the inequality (3.1) with the inequality (1.3) and also the inequality (3.3) with the inequality (1.4), it is easy to observe that the inequality (3.1) is a refinement of the inequality (1.3), and the inequality (3.3) is a refinement of the inequality (1.4) by generalizing the number of terms to n numbers.

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